9 Mathematical Quickies & Trickies

Casting Out Nines

The *casting out of nines* is an ancient and interesting method of checking the accuracy of an arithmetical computation. Any addition, subtraction, or multiplication can be checked by the method of casting out nines.

Consider the multiplication

$$33 \times 11 = 363.$$

To cast the nines out of 33, we add all the digits of the number, obtaining $3 + 3 = 6$. Similarly for 11 we get $1 + 1 = 2$. Casting nines out of 363, we see that $3 + 6 = 9$, so we cast out the 3 and the 6, leaving only the 3.

Now, instead of our original multiplication, we now have a simpler one:

$$6 \times 2 = 3, \text{ since } 6 \times 2 = 12, \text{ and } 1 + 2 = 3.$$

This shows that the original working was indeed correct—the 'digital root' on both sides is 3.

Therefore $33 \times 11 = 363$.

Let's look at another example:

$$2959 \times 59 = 174\ 581$$

By casting out the two nines, the number 2959 becomes $2 + 5 = 7$. Similarly, 59 reduces to 5. Casting out nines or any combination of digits adding up to nine ($4 + 5$; $8 + 1$), the answer 174 581 reduces to 8.

Since the problem has reduced to $7 \times 5 = 35$, and $3 + 5 = 8$, the answer *must* be correct.

DANGER! Casting out 9's is not 100% foolproof!

When a problem has been checked by the method of casting out nines, and the two answers disagree, the problem is necessarily wrong. However, any agreement between the problem and the answer does not necessarily imply that the answer is correct or there is no error in the working.

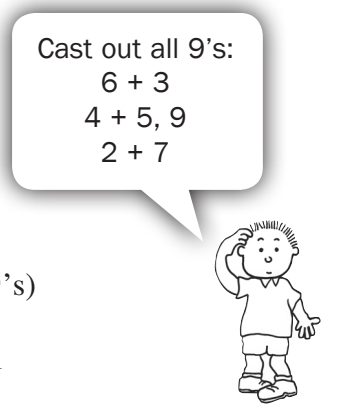
Casting out nines is not a perfect method for detecting any error, no matter how gross. For example, is $32 \times 23 = 529$?

32×23 reduces to 5×5 or 25, and this gives $2 + 5$ which is 7. Casting out nines out of 529 gives $5 + 2$, or 7. There is an agreement between the problem and the answer, yet the method fails to detect the error, because $32 \times 23 = 736$.

More "casting out nines" on addition and subtraction

Example 1

To check whether $3459 + 2575 = 6034$.



Solution:

Add the digits of the sum: $6034 \longrightarrow 4$

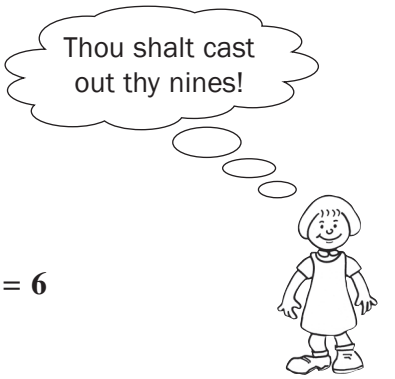
Add the digits of 3459: $(4 + 5); 9 \longrightarrow 3$ (casting out all the 9's)

Add the digits of 2575: $(2 + 7) \longrightarrow 5 + 5 = 10 \longrightarrow 1 + 0 = 1$

Check the final sums: $3 + 1 = 4$

Example 2

To check whether $24 \times 5.38 = 129.12$.



Solution:

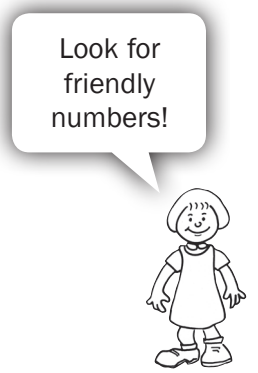
Add the digits of 129.12 (ignoring the 9s): $1 + 2 + 1 + 2 =$ **6**

Add the digits of 24: $2 + 4 = 6$

Add the digits of 5.38: $5 + 3 + 8 = 16 \longrightarrow 1 + 6 = 7$

Check the final digits: $6 \times 7 = 42 \longrightarrow 4 + 2 =$ **6**

1 Mentally calculate: 499 992 – 99 986.

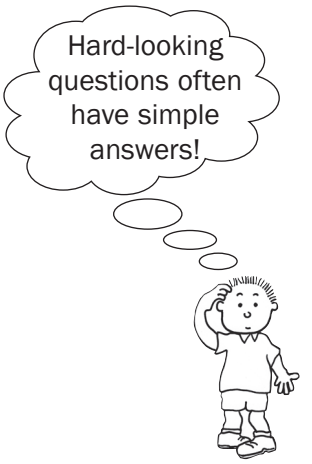


2 What are the next four numbers in this series?

12, 1, 1, 1, 2, 1, 3, ___, ___, ___, ___

3 What is the product of the following series?

$$(x - a)(x - b)(x - c) \ldots (x - z)$$



4 At the half-way of a 1000-km race, Mr Kiasu finds that he has been driving at an average speed of 50 km/h. How fast should he drive the second half of the race so as to achieve an overall average of 100 km/h?

5 To motivate her son to study mathematics, Mrs Kiasu agrees to reward her son 50 cents for every problem solved correctly and to fine him 35 cents for each incorrect solution. At the end of 17 problems, neither owes anything to the other. How many problems did the boy solve correctly?

6 The Armed Forces promises the female officers $100 and a hamper as their rewards for a year. Ms Singa leaves the service after 7 months, and receives the hamper and $20. How much is the hamper worth?

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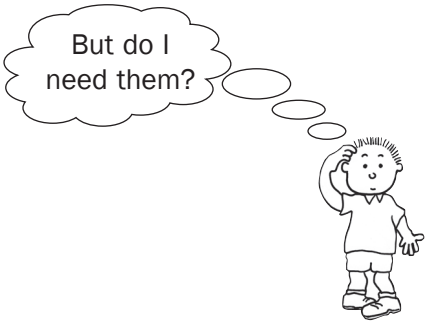
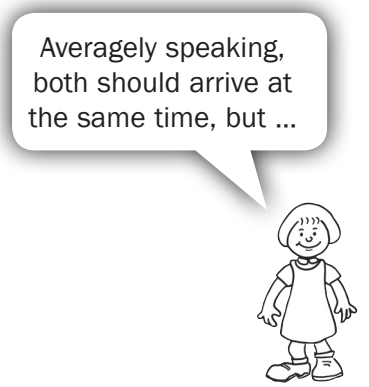
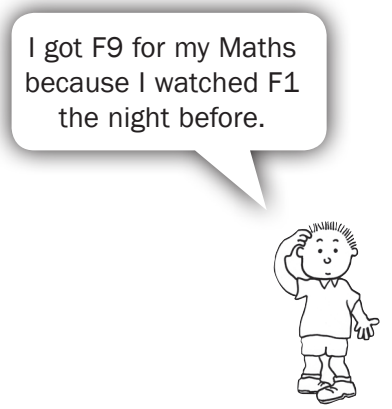


7 Two cyclists leave town *A* for town *B* at the same time, where they stay 4 hours before returning to town *A*. John travels a speed of 30 km/h going and 40 km/h returning. Paul travels 35 km/h each way. Who gets back first?



8 A racing car covered a 6-km track at 140 km/h for 3 km, 168 km/h for 1.5 km, and 210 km/h for 1.5 km. What was the average speed covered for the 6 km?



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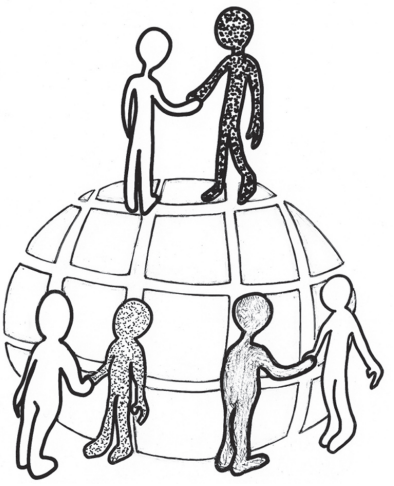
* **9** Flowing down with the current, a man took 4 min to cover 2 km, while against the current, it took him 8 min. How long would it take him to cover the same distance in still water (no current)?

10 Is the number of people in the world who have shaken hands with an odd number of people odd or even?



Peace on Earth